Controlling the arching shape of violin plates to achieve tonal optimisation and repeatability

By Dr. Nigel Harris

This paper builds on my earlier paper (The effect of arching shape on violin sound) and assumes that the reader is familiar with the concepts and terms introduced in that paper. The EAR, rather than the surface arching shape controls the tone of a violin. In this paper I show how a value of EAR can be found that meets the maker’s tonal aspirations. Making violins to a constant value of EAR and using plate thicknesses determined by the criterion of flexural stiffness, can achieve a high degree of tonal repeatability.

How to find the EAR and deviation

The position of the end bouts cross arches
We are going to be measuring the arch height at the end bouts cross arches. Since the long arch slopes in the upper and lower bouts, it is important always to calculate the end bouts cross arch heights at exactly the same positions in the length of the violin. I take the upper bouts cross arch height at a point that is 20% of the body length from the top edge of the plate, and the lower bouts 79% from the top edge. This might not suit all makers but it is sensible to choose a point where the cross arch template will be placed. Whichever place is chosen one must always be consistent and use the same position. It is practical to mark these positions on the long arch templates.

The arching height
Acoustically the arching shape of a violin is the shape of the centreline of the thickness of the wood. If we use the same outside surface shape from one instrument to another, but have differing thicknesses, then the instruments will not have the same arching shape. For convenience, violinmakers measure the arching height from the back of the flat board that the plate is made from. We shall call this the arching height.

But, the effective arching height is found as shown in figure 1. The effective height, \( H \), of an arch, is the height to the crown from the back of the board it is cut from, minus half the thickness at the crown, minus half the thickness at the purfling. The thickness at the purfling is taken as being some approximation of the thickness at the edge. Of course we have no way of telling where the forces in the side arches are centred but the important thing is that the same point is taken in all calculations so that consistency from one violin to another is achieved.

The arching shape
The acoustic effect of an arch cannot be described solely by the height of its crown, since its shape must have some relevance. Consider for example, an end bouts cross arch with a very broad hollow at the sides and then near the centre a steep rise to almost a point, such as one might see in some violins by Nicolo Amati. To measure the arch height to the highest point
would neglect the fact that most of the arch height is much lower. To take this into account I have devised an additional factor called the "shape factor". This is used to modify the measured crown height to take into account not only the shape of the arch but also the way in which it is loaded. The centre bouts cross arch of the belly is loaded at the two points of the bridge feet (minus the sound post load), the centre bouts back arch is single point loaded by the sound post, and the end bouts cross arches are loaded by distributed loads. The way in which I derive the arching shape factors is shown in the Appendix. This added complication of having to find the shape factors for the intended arching shapes, can be avoided by using the same cross arch template shapes for all the instruments made. In this case some nominal shape factors can be used. For this purpose I suggest that the shape factors given below, in the specimen calculation of the EAR, be used.

The shape of a cross arch largely determines its flexibility. It was shown [ref. 1] that the end bouts cross arches will both widen and increase their crown height when a string tension increase is applied. This tends to straighten out the curves in the arch and make it deform slightly to a more straight-line shape from the edge to the centre. A compression in the arch would increase the curves in the arch. This applies to all cross arches, and the shape, which they will tend to deform to, is called a line of thrust.

Suppose one has a walking stick that is curved in its length. If one leans on it, it will bend by buckling. The more curved the stick is, the more it tends to buckle. In the walking stick, the line of thrust is a straight line from the hand to the point where the stick touches the ground. The greater the bend in the stick the more it will buckle. Or put another way, the more the shape of the stick lies away from the line of thrust the more flexible it is. In this paper I am going to refer to the flexibility of an arch shape, which is determined by how much it varies from the line of thrust. The resistance to buckling comes from the flexural stiffness of the wood. In my paper on thicknessing [ref 2], I talked about the flexural stiffness of the wood, which is determined by how thick it is.

The lines of thrust for all cross arches are shown in the appendix. If an arch is built with the maximum difference in shape from the line of thrust shape, it will most easily deform. This could be described as a flexible shape. If the shape closely follows the line of thrust it is an inflexible shape. When the deformation takes place the wood must flex in cross grain bending. The resistance of a violin to being bowed depends (among other things) on how flexible the shape is and how stiff the wood is in bending.

**Calculation of the EAR**

The method used for calculating the EAR is best demonstrated by a specimen calculation for a specific example. We do a calculation like this for every instrument we made. The example given here is taken from our record book. Figure 2 shows the arch heights and wood thicknesses used in this example, measured at the upper, centre and lower bouts. The diagram represents the long arch shapes of the belly and back.
First find the effective arch height, from the height at the crown, minus half the thickness at the crown, minus half the thickness at the plate edge, multiplied by the shape factor. The figure outside the brackets is the shape factor.

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<td></td>
<td><strong>Belly</strong></td>
<td><strong>Back</strong></td>
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<tr>
<td><strong>Upper</strong></td>
<td>(12.50-1.3-1.625) 0.85 =8.139</td>
<td>(10.50-1.5-1.625) 0.8 =5.900</td>
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<td><strong>Centre</strong></td>
<td>(15.38-1.475-1.625) 0.95=11.666</td>
<td>(14.38-2.25-1.625) 1.1=11.555</td>
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<td><strong>Lower</strong></td>
<td>(12.70-1.3-1.625) 0.87 =8.504</td>
<td>(10.46-1.6-1.625) 0.82 =5.933</td>
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Now divide the upper and lower bouts affective arch heights by the centre bouts effective height.

- Upper belly, $\frac{8.139}{11.666}=0.698$
- Upper back, $\frac{5.900}{11.555}=0.511$
- Lower belly, $\frac{8.504}{11.666}=0.729$
- Lower back, $\frac{5.933}{11.555}=0.513$

Now average the upper belly and upper back figures to find the upper bouts EAR and the deviation, and similarly for the lower bouts (expressed as the EAR +or- the deviation).

- For the upper bouts: $(0.698+0.511)/2 = 0.600 +or- 0.089$
- For the lower bouts: $(0.729+0.513)/2 = 0.621 +or- 0.108$

### A strategy for finding a suitable value of EAR

What is a good value for EAR? I suggest that the simplest way of finding a value for the EAR is make a series of violins, and by trial and error, find the optimum value for EAR. I will explain how this can most effectively be done.

Begin by choosing a well-respected violin model suitable for copying the arching shape from. By making a theoretical study using derived arching shape factors, I have found that the model given by Sacconi as being his average of many Stradivaris [ref 3], is close to a good EAR, although it may be a touch high in EAR. The Paganini Cannon is also about right. The “La Messie” Strad is too low in EAR and I would not recommend it. But any model that you might have confidence in can be used, because if it is wrong this will be found out later and can be corrected in the next violin.

Eight templates should now be made. One for the long arch of the belly and one for the long arch of the back (marked with nicks to locate the positions of the cross arches). These should be designed to hook over the bottom edge of the violin at the lower bouts. Great care should be taken in making these templates to ensure that the arching height at the upper bouts, the centre bouts and the lower bouts agrees exactly with the model. Then make half width templates for the belly upper and lower bouts, the back upper and lower bouts and the belly and back centre bouts. All the templates should hook over the plate edges and should sit on the low point of the edge sinking. Templates for the arches near the corners can be taken from the model, or they can be made up by the maker to fit the “as-built” shape of the arching the maker has formed on the first instrument made to the pattern.

First make the outside shape of the back. Using the callipers, carefully measure and record the as-built arching height (measured from the back of the board to the top of the arch) at the centre bouts and end bouts.

Then thickness the back. For the reasons given above, it is important to maintain the same long-grain and cross-grain bending stiffnesses from violin to violin. To do this the reader is referred to the thicknessing method presented in my paper “Plate stiffness as the criterion for determining the thickness of violin plates” [ref. 2].
The next steps are straightforward. Simply make the belly to the arching templates and thickness it using the same method used for the back [ref 2]. Again record accurately the arching heights at the centre and end bouts, and the thicknesses.

We must now calculate the EAR and deviation of the instrument that has been made. Unless it is intended always to use the same cross arch templates, it is necessary to derive the shape factors for the arching using the method set out in the appendix. Makers who might find this difficult can avoid the need to do it by using the shape factors I have used in the specimen calculation given above. The error involved will not matter if the same cross arch templates are used always. The virtue of using the correct shape factors is that the shape of the cross arch templates can be changed at any time and the effect on the EAR can be predicted and allowed for. Calculate the as-built EAR and deviation by the method shown in the specimen calculation.

**Tonal assessment of the EAR**

So, we have made a violin and we know what its EAR and deviation are. We must now decide if this EAR and deviation suit our tonal aspirations. If not we must decide how best to change the EAR so we can make a better violin with the mistakes corrected. Learning from what we have made is fundamental to progressive violinmaking, so this assessment process is very important.

One can, and should, play the violin and think carefully about the sound. The tonal characteristics of both high and low EAR violins have been described in Part 1 [ref 1]. For example, it may be that the EAR is too low in the lower bouts and too high in the upper bouts. This is very often the case if the maker has made the back and belly long arches symmetrical about the centre of the length of the body.

To assist in judging the tonal results, additional tests can be made. If a bridge with the ankles of the feet unusually close, is put on the violin, local bending effects caused by the eccentricity of the bridge feet from the sound post and the bass bar will make the belly centre bouts cross arch behave as though it is more flexible. An alternative way of achieving a similar effect is to move the sound post out closer to the side of the instrument thus creating a similar eccentricity with the bridge foot. Now, if the centre bouts cross arch is more flexible it will lower the C/L ratio by transferring load from the cross arch to the long arch. Now that the instrument has had its C/L lowered, it would require a lower EAR. If the EAR was previously right or too high, the tone will now be made worse and will have the characteristics of EAR too high. If the EAR was previously too low, the tone will have been improved. The converse of this is that if the sound post is put nearer to the centre of the instrument the arch becomes stiffer, attracts load on to the cross arch system, and raises the C/L ratio. This means that the instrument now needs a higher EAR. If the EAR was previously right or too low, the sound will get worse. If the EAR was previously too high, the tone will now improve. Remember that the G string tone is most affected by the EAR of the lower bouts and the A string tone most affected by the EAR of the upper bouts [ref 1]. There is sufficient information here to determine if the EAR of an instrument is either too high or too low, in each of the upper and the lower bouts.

It is widely known that the distance between the bridge feet and distance of the sound post from the centre of the instrument affects the tone. The reason for these effects is given in the last paragraph. It should be noted that the tonal effect of moving the sound post closer to or further from the centreline of the violin will vary depending on the EAR. The current fashion for putting the sound post rather closer to the centreline will benefit violins of too high EAR but will be bad for violins of normal or low EAR, which will become woolly and unclear.

**How to modify the design to alter the EAR and deviation**

Given a clear understanding of how the shape of the body affects the EAR, we can change our model in both the EAR and deviation. For example, if it was decided to lower the EAR but
raise the deviation, this can simply be done by lowering the back end bouts cross arch. If we wanted to raise the EAR and lower the deviation, we can raise the back end bouts cross arches. If we wanted to raise the EAR and the deviation, we simply raise the belly end bouts cross arches. Having decided what to do, redo the calculation of the EAR and deviation making the arching height changes necessary to bring the EAR and deviation to the new values desired. This requires a bit of trial and error.

**Making a violin to a predetermined value of EAR**

Having decided to modify the EAR, we now have the problem of making a violin to a prescribed value of EAR. This is what we do in our workshop for every violin.

Using the new end bouts cross arch heights, modify the shape of the long arch templates. Assuming the changes required are not great the cross arch templates can remain the same. The arch height in the centre and end bouts should be checked by measurement with the callipers rather than relying on the arching template alone. But the arching template is good for the areas in between these points. Make the back to the new shape, and thickness it using the stiffness criterion. Record the as-built arching heights at the centre and end bouts (before hollowing it out) and the thicknesses at the crown heights. It does not matter if there is a slight difference between the intended shape and the as-built shape.

Having built a back, we must now make a belly that will have the shape and thicknesses to give the desired EAR when combined with the back we have made. So we now do the EAR calculation using the figures we have for the back. Put in a figure for the height of the belly centre bouts cross arch. Now guess what the belly thicknesses might be (if the guess is wrong there is a way out, as will be seen). The only unfilled in gap in the calculation is now the belly arch heights in the upper and lower bouts. We can find these figures by calculation.

Now we can make the belly surface shape and record the exact as-built arch heights. The long arch template for the belly need not be altered in shape for every violin made. Just measure the arch heights at the centre and end bouts with the callipers and bring them to the right height. The template should fit well enough to be used for the areas between these key points. The belly should now be thicknessed, using the stiffness criterion method. It was pointed out in the stiffness paper [ref. 2], that the belly thickness at the centre bouts has very little effect on the tone. We can therefore use any thickness at the belly centre, as necessary to bring the EAR to the correct value. While the end bouts thicknesses are controlled by the thicknessing criterion, the centre bouts thickness is controlled by the need to keep the EAR on target. During the thicknessing process of the belly it will be necessary to make a number of calculations to establish the right centre bouts thickness to suit the end bouts thickness. To speed this up we have made up a small computer programme that after all the known variables are entered, will fill in the gaps in the calculation for us.

**Reassessment of EAR and deviation**

Having made our new violin we can again assess the tone. If necessary this can lead to further changes in the EAR. By this process we can soon reach a point where we have an EAR that will give us a repeatable tonal quality with which we are happy. I very rarely have reason to change the EAR I work to.

**The design logic behind some of the well known classical models**

Since getting the EAR correct is very important to ensure the most open pure tone from the violin, it might be interesting to begin by designing a violin that gives us the widest margin for error in the EAR. A large deviation will give us a wide margin for error in the EAR, because the end bouts cross arch heights would have to be more in error to influence the sound. We should also choose a fairly high arch. Now, if the arching is high the EAR will tend to be high. So we are arriving at a design that has high arching, a big difference between the height of the back and belly end bouts cross arches (high deviation), and a high value for
the EAR. This design is quite like the violins of Stainer. The amount of playing required to open a violin up does seem to be related to how much the EAR is wrong. The Stainer model gives the widest possible margin for error in the EAR and a violin of this model could be predicted to take less time to play in. However, because of the high belly end bouts cross arches this design will be weak in the first harmonic, and there is little chance of getting a fat rich sound at the players ear.

Let us look now at how we can modify this design to reduce the belly end bouts cross arch heights, without lowering the deviation or the arching height. If we could do this, it would enable us to keep the widest margin of error for getting the EAR correct. The EAR relates to the C/L ratio, so how can we reduce the C/L ratio? One way is to make the centre bouts cross arch as flexible as possible, thus transferring load to the long arch. The shape of the Stainer centre bouts cross arch is designed to make it very flexible. The arch scoops to a narrow but deep sinking at the edge and then rises very steeply and is rather flat across the top (not dissimilar to the diagrammatic shape shown in figure 3). This maximises the departure from the line of thrust so it is very flexible. This would transfer load to the long arch and lower the C/L ratio and hence the EAR. Thus the belly end bouts cross arches can be at their lowest height while still having a high arch and a high deviation. The model of Nicolo Amati is also based on the principle of fairly high arching and often a high deviation. The centre bouts cross arch is made as flexible as possible by having a very wide scoop at the side and a rise to a high point at the centre (thus giving a big difference from the line of thrust). But still we have the disadvantage of high belly end bouts cross arches. On the “Grand Pattern” instruments we usually see a reduction in the deviation thus lowering the belly end bouts cross arch height.

Stradivari’s answer to this problem was firstly to reduce the arching height. If the arching height is reduced the distance from the arch to the line of thrust is reduced and the arch flexibility is reduced. So the flexibilities of the long arch and the centre bouts cross arch are reduced. The cross arch support system involves the centre bouts cross arch, the side arches and the end arch. The drop in flexibility only applies to the centre bouts cross arch. So the cross arch support is not reduced in flexibility as much as the long arch support. The net effect would be to transfer bridge support from the cross arch to the long arch thus lowering the C/L ratio. Since the EAR varies with the C/L ratio, the EAR must be lowered. So lowering the arching height would bring a disproportionate drop in the end bouts cross arch heights. The drop is disproportionate because lowering the arching not only lowers the end bouts cross arches proportionately but also lowers the EAR, so we get a further drop in the height of the end bouts cross arches. It is clear from looking at classical instruments in general that the lower the arching height is, the lower is the EAR. Strad dropped the deviation from that used by Stainer, although Strad rarely ever used what could be called a low deviation.

But Strad also further increased the flexibility of the centre bouts cross arches. If one looks at the arching of Stainers and Amatis and visualises the line followed by contours around the arching, it can be seen that they swing in at the centre bouts and out at the end bouts. The gives the plates compound curves in the centre bouts. The effect of this is that when the wood tries to flex in the centre bouts cross arches, it meets the resistance caused by this shape. Strad always made the contours through the centre bouts as straight as possible [ref 3], thus reducing the compound curve. A Strad violin belly might be described as two upside down saucers, representing the end bouts, joined by a tapered straight sided barrel vault. This made the centre bouts cross arch more flexible and lowered the C/L ratio, which lowered the EAR, which lowered the belly end bouts cross arches and made the tone sound richer. Having straightened the contour lines at the centre bouts it was clearly logical to alter the old Amati outline shape by straightening the centre bouts outline to match.

Guarneri del Gesu often experimented with the centre bouts cross arches. In general he retained fairly straight contours at the centre bouts. He tried other things including going back to the very flexible shape of cross arch introduced by Nicolo Amati. Anyone looking at
these violins might think they must be very low in the EAR but the centre bouts cross arch
would have a very high shape factor so the EAR would not necessarily be low. He often
reduced the deviation to a point where it was quite low. This would lower the belly end bouts
cross arches and consequently increase the first harmonic sound. Provided the EAR was still
correct there would be very little loss of sound in the higher harmonics. However in his effort
to get the end bouts cross arches as low as possible, he often built his violins with an EAR
that was below the optimum. It was pointed out that violins of optimum EAR are very open
and responsive right from new and the high and low EAR violins are not. However there is
some evidence that the passage of time and years of playing does eventually open the tone up.
Not withstanding this, the tonal characteristics that are determined by the EAR remain with
the violin for all time. Tonally the effect of the low EAR would be to make the lower strings
sound hollow in quality (the metal tank effect), very rich to the player but less open in sound.
When they do open up they have a metallic quality to the brilliance. Of course many del
Gesu violins do not show the characteristics of low EAR violins. When Paganini was
choosing a del Gesu they were still relatively young. The Canon has a greater deviation than
many del Gesu violins and would therefore have been more likely to be open.

Violins are very poor radiators of the first harmonic of the lowest notes on the G string.
There is therefore little point in attempting to get low belly end bouts cross arches on the
lower bouts. The instruments of Stradivari and other great makers usually show a high
deviation in the lower bouts and a lower deviation in the upper bouts. What they are doing
here is to give up on trying to get a strong first harmonic on the G string in the interests of
getting the widest margin of error in the EAR and thus gaining in the brilliance and projection
of the high harmonics. (This difference between the lower and upper bouts deviation can be
seen in both the Stradivari and the del Gesu shown in figure 6, in Part I of this paper.) Anne
Sophie Mutter once said that most Strads would out-carry a del Gesu on the G string, because
of the brilliance of the sound. The higher deviation in the lower bouts of the Strads would be
consistent with that opinion. I suspect most violinists trying both would think the del Gesu
the more powerful because of the big fat sound under the ear. That brings us to the question
of what particular characteristics must violin sound have, in order to carry well.

Making copies

Suppose one makes a violin using templates to control the arching shape accurately, and then
assembles, varnishes and strings it up. After a couple of weeks, try fitting the arching
templates to the instrument. I think most makers would be surprised how different the
arching shape has become. If the arching templates were derived from a particular instrument
or a plaster cast of an instrument, the copy will now be different. Acoustically, the arching
shape is the shape of the centreline of the wood, so unless the wood thicknesses are exactly
the same as the original, the arching cannot be the same, regardless of any distortion that may
have occurred. But even if there is no distortion of the arching and the wood thicknesses are
the same, the cross grain and long grain bending stiffnesses of the plates are unknown and are
unlikely to be the same as that of the original. The inevitable conclusion is that one can never
truly copy an instrument.

My approach has been separately to control the EAR (which is the important tonal
determinant in the arching) and the long-grain and cross-grain bending stiffnesses of the
wood. By this means one can get close to the same sound with every instrument. These
variables can be modified independently of each other to improve the design. Now, of course
the arching templates will not fit the finished instrument either, but that does not matter with
this method of working. Since the arching shape (as defined by the EAR) has been evolved
by trial and error, it will give the desired sound after the distortion has taken place. I believe
that this approach alone can lead to the ability consistently to produce violins of good and
predictable tone.
Conclusions
Tonal repeatability does not require that the arching shape be the same, but rather that the EAR is the same. It is also essential that the long and cross grain bending stiffnesses of the wood are the same. The EAR can be maintained by first making a back, assessing its geometry, and then designing a belly to work with it. The final scope for adjustment of the EAR is made possible by the thicknessing of the belly centre bouts. The method of thicknessing given in my earlier paper [ref 2] will control the long and cross-grain bending stiffnesses of the wood.

References


APPENDIX  The shape factor

The arching shape factor is used to modify the measured crown height to take into account both the shape of the arch and the way in which it is loaded. The standard for comparison for arches is the actual line taken by the forces in the arch. This is called the line of thrust.

Figure 3. Centre belly arch shape compared to the line of thrust, to derive shape factors.

Figure 3(a), shows the line of thrust in a belly centre bouts cross arch with forces applied by the two bridge feet (the sound post contribution having been deducted). If the arch shape was the same as the shape of the line of thrust, there would be only direct forces in the plate without any bending. For the purposes of calculating the EAR the plate will behave as though the top of the cbx arch (centre bouts cross arch) was the height of the line of thrust. But what is the height of the line of thrust? A useful approximation can be made as shown in (b). Draw the CBX arch, and over it draw the line of thrust such that the sum of the two areas “α” is equal to the area “β”. The shape factor for the arch is the height of the line of thrust divided by the height of the arch. s = H/h or H = sh

In figure 4(a), the line of thrust of a back centre bouts cross arch is shown, and for simplicity, shift the load to the centre to produce the symmetrical line of thrust shown in (b) (the resulting final error is surprisingly small). In (c) the actual arch is drawn, and the line of thrust superimposed on it such that the areas cut off above the line equal the areas cut off below the line. The back cbx arch will behave as though it were of height H and this we can find by multiplying the arch height h by the shape factor s.
The ebx arches do not have simple point forces, or loads, like the cbx arches, but have distributed loads. If the load were uniformly distributed, the line of thrust would be of a parabolic form. It is not known exactly how the load is distributed but it must have a bias towards the centre of the span given that it arises from an interaction with the long arch. If the loading distribution is taken to be parabolic, the resulting line of thrust will be a third degree equation. By making a drawing of the shape of the end bouts cross arch and superimposing on this a line of thrust which is a third degree equation, such that the areas cut off above the line of thrust are equal to the areas cut off below the line of thrust, the shape factor as given by $H/h$ can be found.